Stable Local Neural Control of Uncertain Systems

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Abstract

Tracking control of a class of nonlinear uncertain, multi-input, multiple-output systems is addressed in this paper. The control system architecture uses neural networks for function approximation, certainty equivalent control inputs to cancel plant dynamics and smoothed sliding mode control to insure that the trajectories remain bounded. Lyapunov analysis is used to derive equations for the sliding mode control, neural network training, and to show uniform ultimate boundedness of the closed loop system. A simple simulation example is used to illustrate control system performance.

1 Introduction

Control of uncertain, nonlinear systems is a problem that has been approached in a variety of ways. Two of the most common approaches, especially in the area of flight control are: 1) gain scheduled control and 2) adaptive control. Gain scheduled approaches often expend a large amount of effort to model the plant, thus reducing the uncertainties. Adaptive control approaches deal with parametric uncertainties by changing the control characteristics as data is gathered. However, these adaptations are typically made without memory of the events which precipitated the changes. Neural control differs from these two traditional approaches in that it incorporates a means for learning from gathered data and remembering what has been learned.

By neural control we mean control system structures which incorporate neural networks as spatially dependent mappings to affect the commands computed by the control system. Neural control is of much current interest because it provides a means to address control problems without need for large amounts of *a priori* model information, and because it incorporates a *memory* of events that relieves the need for continual re-adaptation. The motivation for the use of neural control systems is the potential of *learning* to control a plant with less effort spent on *a priori* system modeling, with enhanced robustness to gradual temporal changes in the system dynamics, and improved overall performance for systems with state dependent nonlinearities.

Neural control is based on on-line learning which takes the form of weight adjustments based on data as it becomes available. The network must be capable of using the information contained in the data to learn system characteristics. However, one of the difficulties with networks (especially multi-layer perceptrons - MLPs) is that they often learn to better fit data in one area of a domain at the expense of the fit in other regions of the domain. In effect, they forget what was previously learned in an effort to learn new information. This tendency has been referred to as *non-local learning* [1, 11] or *temporal crosstalk* [8] and it's impact in the context of control has also received attention in recent years [1, 2, 3, 8, 9].

In this paper we present a control design approach for a class of nonlinear, affine, Multiple-Input Multiple-Output (MIMO) plants where the number of inputs is the same as the number of outputs. The control objective is to achieve tracking of some desired state trajectories. An adaptive bounding technique is employed to handle the unknown network reconstruction error and sufficient stabilizability conditions on the unknown control multiplier functions are derived. The Lyapunov synthesis approach is used to derive a neural control system with guaranteed stability properties.

2 Problem Formulation

To illustrate the design and analysis approach we consider a two input, two state system where the states are available for measurement. We define the dynamics as

$$\dot{x} = w_1(x,z) + w_2(x,z)u_1^* + w_3(x,z)u_2^* \qquad (1)$$
$$\dot{z} = w_4(x,z) + w_5(x,z)u_1^* + w_6(x,z)u_2^* \qquad (2)$$

where x and z are the states, $(x,z) \in \mathbb{R}^2$, and u_1^*, u_2^* are the control inputs. The dynamics of the system are represented by unknown functions w_1, w_4, and by unknown control multipliers w_2, w_3, w_5, w_6. To prevent loss of stabilizability we assume that each of the control multiplier functions is positive and has known lower bound represented with a subscript L,

$$\begin{array}{ll} w_2(x,z) \geq w_{2L} > 0 & w_3(x,z) \geq w_{3L} > 0 \\ w_5(x,z) \geq w_{5L} > 0 & w_6(x,z) \geq w_{6L} > 0 \end{array}$$

$\forall x, z \in \Omega$, where $\Omega \subset \mathbb{R}^2$ is a domain of interest. For the derivation of controls that will follow, we also assume that two of the control multipliers have known upper bounds defined using a subscript U.

$$w_3(x,z) \leq w_{3U} \quad w_5(x,z) \leq w_{5U} \tag{3}$$

The results shown in this paper can also be shown to hold for the cases where $sign(w_2 w_3) = sign(w_5 w_6) = +1$ or $sign(w_2 w_5) = sign(w_3 w_6) = +1$.

Many flight control systems can be represented by the class of nonlinear systems described in equations (1) and (2). Although aircraft dynamics are inherently nonlinear, flight control system designs have historically relied on linear time invariant models of the form

$$\begin{array}{rcl} \dot{x} & = & Ax + Bu \\ y & = & Cx. \end{array}$$

The matrices (A,B,C) can vary greatly from one flight condition to another, so designs are typically carried out at a number of points in the flight envelope and then blended together. This has worked well because aircraft dynamics are dominantly a function of states such as dynamic pressure and angle of attack. These matrices are very seldom represented as a function of the controls themselves because the controls respond faster than states and control effectiveness (represented by B) *is* predominantly independent of the control input.

Since the scheduling of stability and control derivatives can be very nonlinear, aircraft dynamics require models that are nonlinear functions of the state and affine with respect to the control:

$$\dot{x} = f(x) + g(x)u. \tag{4}$$

Next, we proceed to the design of a stable neural controller for the system described by (1),(2). Each of the two control inputs, u_1^* and u_2^*, in equations (1) and (2), is generated by the control system and consists of two components:

$$u_1^* = u_1 + u_{s1} \tag{5}$$

$$u_2^* = u_2 + u_{s2} \tag{6}$$

where the u_1 and u_2 are certainty equivalent type control inputs and u_{s1} and u_{s2} are sliding mode type control inputs. The control system objective is to have the states track desired reference trajectories, x_d and z_d, which are provided externally.

It is often the case that a control designer has a rough estimate of the characteristics of the plant dynamics either through analytical modeling or thorough empirical studies. The control approach being used here directly allows for this type of information to be used [6, 7], however, for notational simplification, the functions in equation (1) are assumed to be completely unknown.

Each of the unknown functions, w_i, in equation (1) and (2) are modeled using a linearly parameterized combination of Radial Basis Functions (RBFs). The approximations are represented by

$$\hat{w}_i(x,z) = \theta_{w_i}^T \xi(x,z). \tag{7}$$

where ξ are the basis functions and θ_{w_i} is a column vector of parameters. The parameter vector, θ_{w_i}, has k elements where k is the number of basis functions used. We are using the same basis functions for each approximation because it makes the notation simpler, however this is not required.

We define the *best* approximation using

$$\theta_{w_i}^* = \arg \min_{\theta_{w_i} \in \mathbb{R}^k} \{ \sup_{(x,z)\in\Omega} |w_i(x,z) - \theta_{w_i}\xi(x,z)| \} \tag{8}$$

and

$$w_i^*(x,z) = \theta_{w_i}^* \xi(x,z). \tag{9}$$

We also define the parameter estimation error as $\tilde{\theta}_{w_i} = \theta_{w_i} - \theta_{w_i}^*$.

Unless the actual functions, w_i are linear combinations of the ξ basis functions, there will be errors remaining in each approximation after finding the *best* θ_{w_i} vector. The error which remains after the best fit has been achieved is referred to as the *reconstruction error* and is given by

$$\delta_{w_i}(x,z) = w_i(x,z) - w_i^*(x,z). \tag{10}$$

To remove the necessity to assume *a priori* knowledge of a bound on the reconstruction error, we develop an adaptive bounding scheme where the bound on $\delta_{w_i}(x,z)$ is estimated on-line. Let

$$\psi_{w_i}^* = \sup_{(x,z)\in\Omega} |\delta_{w_i}(x,z)| \tag{11}$$

be the unknown bound on the reconstruction error. The unknown parameter bound estimation error, $\tilde{\psi}_{w_i}(t)$, is defined as

$$\tilde{\psi}_{w_i}(t) = \psi_{w_i}(t) - \bar{\psi}_{w_i}^* \tag{12}$$

where $\bar{\psi}_{w_i}^* := \max\{\psi_{w_i}^*, \psi_{w_i}^o\}$ and $\psi_{w_i}(t)$ is the on-line estimate of the bound. The constant $\psi_{w_i}^o$ is a design parameter that will appear in the adaptation law for updating $\psi_{w_i}(t)$.

Now we define the tracking errors

$$e_x = x - x_d \tag{13}$$
$$e_z = z - z_d, \tag{14}$$

which are used to define a sliding mode surface vector as

$$\bar{s} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} e_x + c_1 \int e_x d\tau \\ e_z + c_2 \int e_z d\tau \end{bmatrix} \tag{15}$$

where c_1 and c_2 are positive design constants. By defining the sliding mode quantity in this way, we have a vector of filtered errors which induce integral control action [5, 10].

The problem we have described is tracking control for a MIMO, affinely represented model with parametric uncertainty due to unknown $\theta^*_{w_i}$, and bounding uncertainty due to the unknown reconstruction error bounds, $\psi^*_{w_i}$. The control design approach followed in this paper is to use neural networks to approximate the unknown system nonlinearities w_i, vector sliding mode control with adaptive bounding to insure boundedness, and integral action to improve transient response.

3 Neural Control Design

Consider the Lyapunov function

$$V = \frac{1}{2}s^T s + \sum_{i=1}^{6} \frac{\tilde{\theta}^T_{w_i}\tilde{\theta}_{w_i}}{2\gamma} + \sum_{i=1}^{6} \frac{\tilde{\psi}^2_{w_i}}{2\gamma} \tag{16}$$

where $\gamma > 0$ is the adaptive gain.

Taking the derivative of equation (16) and substituting from equations (13), (14), and (15) we get

$$\begin{aligned}\dot{V} = s_1\{\dot{x} - \dot{x}_d + c_1(x - x_d)\} + s_2\{\dot{z} - \dot{z}_d \\ + c_2(z - z_d)\} + \sum_{i=1}^{6}\{\frac{\tilde{\theta}^T_{w_i}\dot{\theta}_{w_i}}{\gamma} + \frac{\tilde{\psi}_{w_i}\dot{\psi}_{w_i}}{\gamma}\}.\end{aligned} \tag{17}$$

Using the control equations (5) and (6), the dynamics from equations (1) and (2) and adding and subtracting some terms which sum to zero, we can write the Lyapunov derivative as

$$\begin{aligned}\dot{V} = {} & s_1\{(w_1 + w_2u_1 + w_3u_2 + w_2u_{s1} \\ & + w_3u_{s2}) - \dot{x}_d + c_1e_x \\ & + (\hat{w}_2u_1 + \hat{w}_3u_2) - (\hat{w}_2u_1 + \hat{w}_3u_2)\} \\ & + s_2\{(w_4 + w_5u_1 + w_6u_2 + w_5u_{s1} \\ & + w_6u_{s2}) - \dot{z}_d + c_2e_z \\ & + (\hat{w}_5u_1 + \hat{w}_6u_2) - (\hat{w}_5u_1 + \hat{w}_6u_2)\} \\ & + \sum_{i=1}^{6}\{\frac{\tilde{\theta}^T_{w_i}\dot{\theta}_{w_i}}{\gamma} + \frac{\tilde{\psi}_{w_i}\dot{\psi}_{w_i}}{\gamma}\}\end{aligned} \tag{18}$$

where the functional dependencies of w_i and $\hat{w}_i$ on x, z, and θ_{w_i} have not been written in order to streamline the notation.

We consider a certainty equivalent type control law which can be found by solving the following two simultaneous equations for u_1 and u_2.

$$\hat{w}_2u_1 + \hat{w}_3u_2 = -s_1 - \hat{w}_1 + \dot{x}_d - c_1e_x \tag{19}$$
$$\hat{w}_5u_1 + \hat{w}_6u_2 = -s_2 - \hat{w}_4 + \dot{z}_d - c_2e_z \tag{20}$$

The first terms on the right hand side help make the Lyapunov derivative negative, the second terms attempt to cancel the unknown nonlinearities in the Lyapunov derivative, and the last two terms subtract out terms that are known. Using (19) and (20) in equation (18), and regrouping we obtain

$$\begin{aligned}\dot{V} = {} & s_1\{(w_1 - w_1^*) + (w_1^* - \hat{w}_1) + [(w_2 - w_2^*) \\ & + (w_2^* - \hat{w}_2)]u_1 + [(w_3 - w_3^*) + (w_3^* - \hat{w}_3)]u_2 \\ & + w_2u_{s1} + w_3u_{s2} - s_1\} \\ & s_2\{(w_4 - w_4^*) + (w_4^* - \hat{w}_4) + [(w_5 - w_5^*) \\ & + (w_5^* - \hat{w}_5)]u_1 + [(w_6 - w_6^*) + (w_6^* - \hat{w}_6)]u_2 \\ & + w_5u_{s1} + w_6u_{s2} - s_2\} \\ & + \sum_{i=1}^{6}\{\frac{\tilde{\theta}^T_{w_i}\dot{\theta}_{w_i}}{\gamma} + \frac{\tilde{\psi}_{w_i}\dot{\psi}_{w_i}}{\gamma}\}\end{aligned} \tag{21}$$

Using the reconstruction error bounds of equation (11) we can write a bound on the Lyapunov derivative as

$$\begin{aligned}\dot{V} \leq {} & \bar{\psi}^*_{w_1}|s_1| + \bar{\psi}^*_{w_2}|s_1u_1| + \bar{\psi}^*_{w_3}|s_1u_2| \\ & - \tilde{\theta}^T_{w_1}\xi s_1 - \tilde{\theta}^T_{w_2}\xi s_1u_1 - \tilde{\theta}^T_{w_3}\xi s_1u_2 + s_1w_2u_{s_1} \\ & + s_1w_3u_{s_2} - s_1^2 + \bar{\psi}^*_{w_4}|s_2| + \bar{\psi}^*_{w_5}|s_2u_1| \\ & + \bar{\psi}^*_{w_6}|s_2u_2| - \tilde{\theta}^T_{w_4}\xi s_2 - \tilde{\theta}^T_{w_5}\xi s_2u_1 \\ & - \tilde{\theta}^T_{w_6}\xi s_2u_2 + s_2w_5u_{s_1} + s_2w_6u_{s_2} \\ & - s_2^2 + \sum_{i=1}^{6}\{\frac{\tilde{\theta}^T_{w_i}\dot{\theta}_{w_i}}{\gamma} + \frac{\tilde{\psi}_{w_i}\dot{\psi}_{w_i}}{\gamma}\}\end{aligned} \tag{22}$$

The inequality in (22) includes terms which are positive and which include unknown reconstruction error bounds $\bar{\psi}^*_{w_i}$. Also included are terms involving sliding mode controls u_{s1} and u_{s2}. Our objective in defining the sliding mode control is to cancel the positive terms using the estimates of the reconstruction error bounds, ψ_{w_i}. However, we want to insure that the terms involving sliding mode controls are as small as possible to maintain the bound on the Lyapunov derivative. Thus, we define the sum of the four sliding mode controls as

$$\begin{aligned}& [s_1w_2u_{s_1} + s_1w_3u_{s_2} + s_2w_5u_{s_1} + s_2w_6u_{s_2}] \leq \\ & \quad -s_1\psi_{w_1}\tanh(\frac{s_1}{\epsilon}) - s_1u_1\psi_{w_2}\tanh(\frac{s_1u_1}{\epsilon}) \\ & \quad -s_1u_2\psi_{w_3}\tanh(\frac{s_1u_2}{\epsilon}) - s_2\psi_{w_4}\tanh(\frac{s_2}{\epsilon}) \\ & \quad -s_2u_1\psi_{w_5}\tanh(\frac{s_2u_1}{\epsilon}) \\ & \quad -s_2u_2\psi_{w_6}\tanh(\frac{s_2u_2}{\epsilon})\end{aligned} \tag{23}$$

where the inequality must hold for any value that the uncertain functions, w_2, w_3, w_5, and w_6, may take on. Next, we will show how the sliding mode controls which satisfy inequality (23) help achieve the Lyapunov derivative properties that we desire. After the Lyapunov analysis we will return to (23) and show how to solve for the individual sliding mode control components, u_{s1} and u_{s2}.

Substituting (23) into (22) we obtain

$$\begin{aligned}\dot{V} \leq\ & \bar{\psi}^*_{w_1}|s_1| - s_1\psi_{w_1}\tanh(\frac{s_1}{\varepsilon}) + \bar{\psi}^*_{w_2}|s_1u_1| \\ & -s_1u_1\psi_{w_2}\tanh(\frac{s_1u_1}{\varepsilon}) + \bar{\psi}^*_{w_3}|s_1u_2| \\ & -s_1u_2\psi_{w_3}\tanh(\frac{s_1u_2}{\varepsilon}) + \bar{\psi}^*_{w_4}|s_2| \\ & -s_2\psi_{w_4}\tanh(\frac{s_2}{\varepsilon}) + \bar{\psi}^*_{w_5}|s_2u_1| \\ & -s_2u_1\psi_{w_5}\tanh(\frac{s_2u_1}{\varepsilon}) + \bar{\psi}^*_{w_6}|s_2u_2| \\ & -s_2u_2\psi_{w_6}\tanh(\frac{s_2u_2}{\varepsilon}) - \tilde{\theta}^T_{w_1}\xi s_1 - \tilde{\theta}^T_{w_2}\xi s_1u_1 \\ & -\tilde{\theta}^T_{w_3}\xi s_1u_2 - \tilde{\theta}^T_{w_4}\xi s_2 - \tilde{\theta}^T_{w_5}\xi s_2u_1 - \tilde{\theta}^T_{w_6}\xi s_2u_2 \\ & -s_1^2 - s_2^2 + \sum_{i=1}^{6}\{\frac{\tilde{\theta}^T_{w_i}\dot{\theta}_{w_i}}{\gamma} + \frac{\tilde{\psi}_{w_i}\dot{\psi}_{w_i}}{\gamma}\} \qquad (24)\end{aligned}$$

From equation (12) we know that $\psi_{w_i}(t) = \tilde{\psi}_{w_i}(t) + \bar{\psi}^*_{w_i}$. Making this substitution in (24) and regrouping terms we can write

$$\begin{aligned}\dot{V} \leq\ & -s_1^2 - s_2^2 + \sum_{i=1}^{3}\{\bar{\psi}^*_{w_i}[|s_1q| - s_1q\tanh(\frac{s_1q}{\varepsilon})] \\ & +\frac{\tilde{\theta}^T_{w_i}}{\gamma}[\dot{\theta}_{w_i} - \gamma\xi s_1q] \\ & +\frac{\tilde{\psi}^T_{w_i}}{\gamma}[\dot{\psi}_{w_i} - \gamma s_1q\tanh(\frac{s_1q}{\varepsilon})]\} \\ & +\sum_{i=4}^{6}\{\bar{\psi}^*_{w_i}[|s_2q| - s_2q\tanh(\frac{s_2q}{\varepsilon})] \\ & +\frac{\tilde{\theta}^T_{w_i}}{\gamma}[\dot{\theta}_{w_i} - \gamma\xi s_2q] \\ & +\frac{\tilde{\psi}^T_{w_i}}{\gamma}[\dot{\psi}_{w_i} - \gamma s_2q\tanh(\frac{s_2q}{\varepsilon})]\} \qquad (25)\end{aligned}$$

where $q = 1$ for $i = 1, 3$, $q = u_1$ for $i = 2, 5$, and $q = u_2$ for $i = 3, 6$.

To bound the terms involving $\bar{\psi}^*_i$, we use a property of the hyperbolic tangent [6] according to which for any $\eta \in \mathbb{R}$ and any constant $\epsilon > 0$

$$|\eta| - \eta\tanh(\frac{\eta}{\epsilon}) \leq \kappa\epsilon \qquad (26)$$

where $\kappa = .2786$.

The Lyapunov analysis yields the adaptive laws for θ_{w_i} and ψ_{w_i}, which are given by

$$\begin{aligned}\dot{\theta}_{w_i} &= \gamma\{\xi s_jq - \sigma(\theta_{w_i} - \theta^o_{w_i})\} & (27)\\ \dot{\psi}_{w_i} &= \gamma\{s_jq\tanh(\frac{s_jq}{\epsilon}) - \sigma(\psi_{w_i} - \psi^o_{w_i})\} & (28)\end{aligned}$$

where $j = 1$ for $i = 1, 2, 3$, $j = 2$ for $i = 4, 5, 6$, and q is defined as before. The small positive constant, σ, is a design variable which introduces a leakage term into the adaptive law for the purpose of guaranteeing the boundedness of the parameter estimates. The terms $\theta^o_{w_i}$ and $\psi^o_{w_i}$ are also design constants that represent initial estimates of the unknown parameters $\theta^*_{w_i}$ and $\psi^*_{w_i}$ respectively. In the absence of any such *a priori* information, $\theta^o_{w_i}$ and $\psi^o_{w_i}$ can be set to zero.

Using (26) and substituting equations (27) and (28) into (25), the Lyapunov derivative bound is given by

$$\begin{aligned}\dot{V} \leq\ & -s_1^2 - s_2^2 + \sum_{i=1}^{6}\{\kappa\epsilon\bar{\psi}_{w_i} - \sigma\tilde{\theta}^T_{w_i}(\theta_{w_i} - \theta^o_{w_i})\} \\ & -\sum_{i=1}^{6}\{\sigma\tilde{\psi}_{w_i}(\psi_{w_i} - \psi^o_{w_i})\}. \qquad (29)\end{aligned}$$

The Lyapunov bound given by (29) can be written as

$$\dot{V} \leq -cV + \lambda \qquad (30)$$

where $c = \min(2, \sigma\gamma)$ and

$$\begin{aligned}\lambda =\ & \sum_{i=1}^{6}\{\kappa\epsilon\bar{\psi}^*_{w_i} + \frac{\sigma}{2}[|\theta^*_{w_i} - \theta^o_{w_i}|^2 \\ & +|\psi^*_{w_i} - \psi^o_{w_i}|^2]\}.\end{aligned}$$

Although $\dot{V}$ is not negative semi-definite, it is clear from (30) that V is bounded. Since the Lyapunov function was defined as the sum of quadratic sliding mode and parameters error terms, we can conclude that the sliding mode values and parameter errors are all bounded. If we define $\mu > \sqrt{2\frac{\lambda}{c}}$ and use inequality (30), the magnitude of $s_1(t)$ and $s_2(t)$ can be shown to be bounded by μ [6].

The stability properties just described are dependent on the existence of sliding mode controls which satisfy inequality (23). One way to solve for controls, u_{s1} and u_{s2}, is given below.

We first separate (23) into

$$\begin{aligned}s_1w_2u_{s_1} + s_1w_3u_{s_2} \leq\ & -s_1\psi_{w_1}\tanh(\frac{s_1}{\epsilon}) \\ & -s_1u_1\psi_{w_2}\tanh(\frac{s_1u_1}{\epsilon}) \\ & -s_1u_2\psi_{w_3}\tanh(\frac{s_1u_2}{\epsilon}) \qquad (31)\end{aligned}$$

and

$$\begin{aligned}s_2w_5u_{s_1} + s_2w_6u_{s_2} \leq\ & -s_2\psi_{w_4}\tanh(\frac{s_2}{\epsilon}) \\ & -s_2u_1\psi_{w_5}\tanh(\frac{s_2u_1}{\epsilon}) \\ & -s_2u_2\psi_{w_6}\tanh(\frac{s_2u_2}{\epsilon}). \qquad (32)\end{aligned}$$

Next we define the controls as

$$u_{s_1} = -K_1 \tanh(\frac{s_1}{\epsilon}) - K_2 u_1 \tanh(\frac{s_1 u_1}{\epsilon}) - K_3 u_2 \tanh(\frac{s_1 u_2}{\epsilon}) \quad (33)$$

$$u_{s_2} = -K_4 \tanh(\frac{s_2}{\epsilon}) - K_5 u_1 \tanh \frac{s_2 u_1}{\epsilon} - K_6 u_2 \tanh(\frac{s_2 u_2}{\epsilon}) \quad (34)$$

where K_i, for $i = 1, ...6$, are gains to be determined. Substituting (33) and (34) into (31) and (32) we can write

$$\begin{aligned} -s_1\{w_2 K_1 \tanh(\frac{s_1}{\epsilon}) + u_1 w_2 K_2 \tanh(\frac{s_1 u_1}{\epsilon}) \\ +u_2 w_2 K_3 \tanh(\frac{s_1 u_2}{\epsilon}) + w_3 K_4 \tanh(\frac{s_2}{\epsilon}) \\ +u_1 w_3 K_5 \tanh(\frac{s_2 u_1}{\epsilon}) + u_2 w_3 K_6 \tanh(\frac{s_2 u_2}{\epsilon})\} \leq \\ -s_1\{\psi_{w_1} \tanh(\frac{s_1}{\epsilon}) + u_1 \psi_{w_2} \tanh(\frac{s_1 u_1}{\epsilon}) \\ +u_2 \psi_{w_3} \tanh(\frac{s_1 u_2}{\epsilon})\} \end{aligned} \quad (35)$$

and

$$\begin{aligned} -s_2\{w_5 K_1 \tanh(\frac{s_1}{\epsilon}) + u_1 w_5 K_2 \tanh(\frac{s_1 u_1}{\epsilon}) \\ +u_2 w_5 K_3 \tanh(\frac{s_1 u_2}{\epsilon}) + w_6 K_4 \tanh(\frac{s_2}{\epsilon}) \\ +u_1 w_6 K_5 \tanh(\frac{s_2 u_1}{\epsilon}) + u_2 w_6 K_6 \tanh(\frac{s_2 u_2}{\epsilon})\} \leq \\ -s_2\{\psi_{w_4} \tanh(\frac{s_2}{\epsilon}) + u_1 \psi_{w_5} \tanh(\frac{s_2 u_1}{\epsilon}) \\ +u_2 \psi_{w_6} \tanh(\frac{s_2 u_2}{\epsilon})\}. \end{aligned} \quad (36)$$

To illustrate the procedure we consider the terms related to the gains K_1 and K_4. Combining terms in (35), it can be seen that K_1 and K_4 need to be selected such that the following inequality is satisfied.

$$-s_1 w_2 K_1 \tanh(\frac{s_1}{\epsilon}) - s_1 w_3 K_4 \tanh(\frac{s_2}{\epsilon}) \leq -s_1 \psi_{w_1} \tanh(\frac{s_1}{\epsilon}) \quad (37)$$

Similarly using (36) we obtain

$$-s_2 w_5 K_1 \tanh(\frac{s_1}{\epsilon}) - s_2 w_6 K_4 \tanh(\frac{s_2}{\epsilon}) \leq -s_2 \psi_{w_4} \tanh(\frac{s_2}{\epsilon}). \quad (38)$$

We seek gains, K_1 and K_4, that will satisfy (37) and (38) for all evaluations of the functions, w_2, w_3, w_5, and w_6. If we divide all terms in (37) by $s_1 \tanh(\frac{s_1}{\epsilon})$ and all the terms in (38) by $s_2 \tanh(\frac{s_2}{\epsilon})$ we get

$$-w_2 K_1 - w_3 K_4 R \leq -\psi_{w_1} \quad (39)$$

$$\frac{-w_5 K_1}{R} - w_6 K_4 \leq -\psi_{w_4} \quad (40)$$

where R is defined by

$$R = \frac{\tanh(\frac{s_2}{\epsilon})}{\tanh(\frac{s_1}{\epsilon})}. \quad (41)$$

Since we don't know the sign of R, we solve for the worst case bounds for K_1 and K_4.

$$K_1 \geq \frac{\psi_{w_1} + w_3 K_4 |R|}{w_2} \quad (42)$$

$$K_4 \geq \frac{\psi_{w_4} + \frac{w_5 K_1}{|R|}}{w_6} \quad (43)$$

We can see from (42) and (43) that the gains, K_1 and K_4, are at their largest values when w_3 and w_5 take on their largest values and when w_2 and w_6 take on their smallest values. Since it is sufficient that equality applies to (42) and (43), we solve to find the gains.

$$K_1 = \{1 - \frac{w_{5U} w_{3U}}{w_{2L} w_{6L}}\}^{-1}\{\frac{\psi_{w_1}}{w_{2L}} + \frac{w_{3U} \psi_{w_4} |R|}{w_{2L} w_{6L}}\} \quad (44)$$

$$K_4 = \{1 - \frac{w_{5U} w_{3U}}{w_{2L} w_{6L}}\}^{-1}\{\frac{\psi_{w_4}}{w_{6L}} + \frac{w_{5U} \psi_{w_1}}{w_{2L} w_{6L} |R|}\} \quad (45)$$

Similar equations can be written for gains K_2, K_5, K_3 and K_6.

It may be seen that the inverse in equations (44) and (45) can be undefined if $w_{5U} w_{3U} = w_{2L} w_{6L}$. However, if we restrict our attention to plants where

$$\det \begin{bmatrix} w_2(x,z) & w_3(x,z) \\ w_5(x,z) & w_6(x,z) \end{bmatrix} \neq 0 \quad \forall (x,z) \in \mathbb{R}^2, \quad (46)$$

then equations (44) and (45) are well defined. This restriction, (46), which is a stabilizability condition, will hold for plants where u_1 has the primary control authority in (1) and u_2 has the primary control authority in (2). It should also be noted that the gains $K_{1...6}$ are functions of the s_1 and s_2 values and therefore an implicit function of time.

4 Simulation Example

To illustrate the neural control system design, a model has been extracted from an aircraft simulation and modified to provide some nonlinear characteristics while still satisfying the restriction given by (46). The *truth* model is given by

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -.839 & 1.000 \\ -2.556 & -1.696 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} .00208 & w_3(x) \\ w_5(x) & .02 \end{bmatrix} \begin{bmatrix} u_1^* \\ u_2^* \end{bmatrix} \quad (47)$$

The equations of motion given by (47) can be written as

$$\begin{aligned} \dot{x} &= w_{1x}(x) + w_{1z}(z) + w_2(x) u_1^* + w_3(x) u_2^* \\ \dot{z} &= w_{4x}(x) + w_{4z}(z) + w_5(x) u_1^* + w_5(x) u_2^* \end{aligned} \quad (48)$$

where $w_{1x}(x) = -0.839x$, $w_{1z}(z) = z$, $w_2(x) = .00208$, $w_3(x) = .02 + .0012x^2$, $w_{4x}(x) = -2.556x$, $w_{4z}(z) =$

$-1.696z$, $w_5(x) = .13 - .0017x^2$, $w_6(x) = .02$ are all unknown functions.

The neural network approximations use 32 RBF basis functions in an Multi-Resolution Analysis [4] structure on a domain of $[-8, 8]$. To illustrate the learning performance of the neural network the simulations were initialized with *poor* approximations of the truth functions and as the simulation progressed the approximations become closer to the true functions. This resulted in larger control inputs from the certainty equivalent controls and smaller control inputs from the sliding mode. As the network performance improved, the tracking performance improved as can be seen in Figure 1 where the

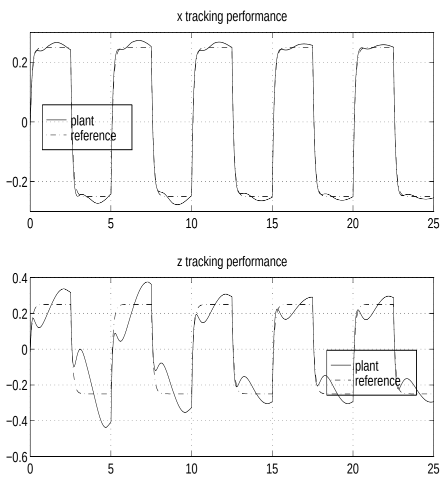

Figure 1: Tracking Performance of the states x(t) and z(t) with respect to the desired trajectories x_d and z_d

dash-dot lines indicate the desired trajectories and the solid lines are the system states.

The function approximation accuracy improves during the simulation as can be seen in Figure 2 where the dashed line is the true function, the solid line is the initialization, and the dash-dot lines are the approximations after the simulation has completed. Note that the function approximation changes are most obvious in the region of the state space where the system was forced to track.

5 Conclusion

A neural control approach for a class of nonlinear MIMO systems has been developed. Lyapunov function analysis been used to insure bounded tracking errors and robust adaptive methods prevent parameter drift. Results have been illustrated with a simple MIMO example.

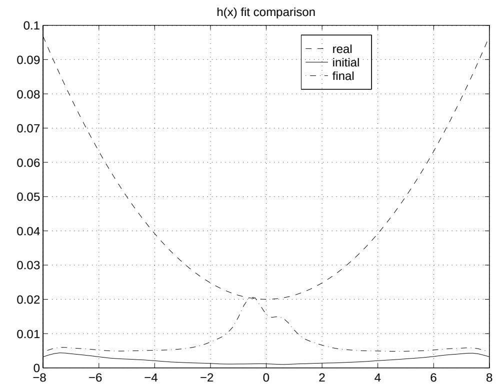

Figure 2: Network approximation of the unknown function $w_3(x)$

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