Satellite Formationkeeping Control in the Presence of Gravity Perturbations

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Abstract

The performance of linear control laws for satellite formationkeeping in the presence of gravity perturbations is considered. Control using a linear quadratic regulator to minimize the error between the actual and desired relative satellite motion is assessed for its ability to maintain a particular formation geometry in the presence of the earth oblateness gravity perturbation. The desired formation geometry is based on the solution to the linear, unperturbed relative motion equations. Specifically, a formation of satellites is chosen whose projected motion onto the earth's tangential plane is a one kilometer circle. It is shown that the linear control laws maintain the formation within error bounds in the presence of gravity perturbations. Furthermore, simulations provide estimates of the maneuvering propellant required to maintain such a formation, providing a baseline for future studies involving formations based on less taxing desired trajectories and more carefully chosen control strategies.

1 Introduction

Flying a close formation of several satellites for distributed space operations has recently become a topic of significant interest. The idea of using several small, unconnected, coorbiting satellites to form a sparse aperture rather than using a single monolithic satellite having a single filled aperture has many advantages as well as many technical challenges. The resolution of an imaging sensor is limited by the maximum achievable baseline or aperture size, so that a single filled aperture sensor is limited by physical size and weight considerations. Meanwhile, larger baselines can be achieved using a satellite formation to form a large sparse aperture simply by increasing the formation size. Hence, the formation is capable of much finer resolution than would be possible otherwise. In addition, smaller satellites are much easier to put into orbit, and so save on launch costs. Finally, since the distributed satellites can be reconfigured into formations of different geometries, a single formation of satellites can be made to perform a variety of different missions.

The first part of the formationkeeping problem is to find a formation geometry that meets the requirements of the desired mission and requires a reasonable amount of maneuvering propellent to overcome disturbances and perturbations in maintaining the formation geometry. Formations based on the Clohessy-Wiltshire, or Hill's, equations [3], which yield closed periodic solutions for the linear, unperturbed relative motion, provide a starting point. Such formations would maintain their geometry indefinitely in the absence of perturbations and disturbances. These formations have been considered by [8].

The high order orbital dynamics include significant perturbations, the most important difference being the earth gravity harmonics. The first harmonic, the J_2 or earth oblateness term, causes the largest perturbation in the motion, and is therefore the most important term to consider. Orbits that are closed in the presence of the J_2 perturbation have also been considered in [9], although these may not be feasible for all missions. In general, closed orbits may not be available for a specific mission, and in any case will not consider all the gravitational perturbations, so that feedback control will always be required.

After a suitable formation has been designed, the optimal control law that uses an optimal amount of propellant for that formation must be found. Linear quadratic control laws have been considered by several researchers [1, 4, 7, 12, 14]. In these cases, the unperturbed dynamics were discretized and discrete-time linear quadratic regulators were found to stabilize the relative motion dynamics and provide formationkeeping for simple formations. Other control laws have been considered in [5, 6, 10, 11, 13].

The goal of this paper is to apply a control strategy based on the linear relative motion or Hill's equations to a particular satellite formation and evaluate the performance in the presence of gravity perturbations. The formation considered will consist of a ring of satellites whose projection onto the tangential plane is a one kilometer circle. Finally, the amount of maneuvering propellant required will be assessed to provide a baseline for future studies, where both better desired trajectories and more advanced control strategies will be considered.

2 Relative Motion Dynamics

The dynamics of a satellite formation can be described in terms of the relative positions and velocities of the constituent satellites. The description of motion is most convenient when it is referred to a moving frame attached to one of the satellites in the formation. The equations of relative motion between two orbiting satellites were derived initially to solve the problem of orbital rendezvous, in which a satellite was to be assembled in orbit and the components were required to rendezvous in sequence for assembly [3]. These same equations are also quite useful in the design of satellite formation geometries and control laws to maintain these formations. In this section we will review the relative motion dynamics of two satellites orbiting close together and give a brief description of their derivation.

Consider two satellites in orbit around a spherical earth. Let the first satellite have position $\bar{r}$ relative to the center of the earth and let the second have position $\bar{\rho}$ relative to the first so that its position relative to the earth's center is $\bar{r}+\bar{\rho}$. The motions of the satellites are given by

$$\ddot{\bar{r}} + \frac{\mu}{r^3}\bar{r} = F_l,$$

$$\ddot{\bar{r}} + \ddot{\bar{\rho}} + \frac{\mu}{|\bar{r}+\bar{\rho}|^3}(\bar{r}+\bar{\rho}) = F_f,$$

where μ is the earth's gravitational constant, $r = |\bar{r}|$, and F_l and F_f are the external disturbances and perturbations on the leader and follower satellites, respectively. Taking the difference of these equations yields

$$\ddot{\bar{\rho}} + \frac{\mu}{|\bar{r}+\bar{\rho}|^3}(\bar{r}+\bar{\rho}) - \frac{\mu}{r^3}\bar{r} = F_f - F_l.$$

Now, assume that the leader satellite is in a circular orbit so that r is constant and that there are no external disturbances or perturbations acting on the satellites. Consider a moving coordinate system attached to the leader satellite where $\hat{\imath}$ is in the radial direction, $\hat{\jmath}$ is in the direction of the motion, and $\hat{k}$ is normal to the orbital plane. Letting the relative position vector be written as $\rho = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and resolving the vector relative motion equation above into components yields

$$\begin{aligned} \ddot{x} - 2\omega\dot{y} - \omega^2 x + \frac{\mu(r+x)}{r^3 d} - \frac{\mu}{r^3} r &= 0, \\ \ddot{y} + 2\omega\dot{x} - \omega^2 y + \frac{\mu y}{r^3 d)^{3/2}} &= 0, \\ \ddot{z} + \frac{\mu z}{r^3 d} &= 0, \end{aligned}$$

where

$$d = \left(1 + \frac{2x}{r} + \frac{x^2+y^2+z^2}{r^2}\right)^{3/2}.$$

Assuming that the distance between the satellites is small and noting that $\omega^2 = \frac{\mu}{r^3}$, the equations can be linearized to yield

$$\ddot{x} - 2\omega\dot{y} - 3\omega^2 x = 0, \quad (1)$$

$$\ddot{y} + 2\omega\dot{x} = 0, \quad (2)$$

$$\ddot{z} + \omega^2 z = 0. \quad (3)$$

These are the Clohessy-Wiltshire or Hill's equations of relative orbital motion [3].

The closed-form solution to Hill's equations (1)-(3) is given by

$$\begin{aligned} x(t) &= \frac{\dot{x}(0)}{\omega}\sin\omega t - \left(3x(0) + \frac{2\dot{y}(0)}{\omega}\right)\cos\omega t \\ &\quad + \left(4x(0) + \frac{2\dot{y}(0)}{\omega}\right), \quad (4) \\ y(t) &= \frac{2\dot{x}(0)}{\omega}\cos\omega t + \left(6x(0) + \frac{4\dot{y}(0)}{\omega}\right)\sin\omega t \\ &\quad - (6\omega x(0) + 3\dot{y}(0))\,t - \frac{2\dot{x}(0)}{\omega} + y(0), \quad (5) \\ z(t) &= \frac{\dot{z}(0)}{\omega}\sin\omega t + z(0)\cos\omega t. \quad (6) \end{aligned}$$

Noting that the third term of $y(t)$ grows unbounded as time increases, the constraint $2\omega x(0) + \dot{y}(0) = 0$ enforces that the follower does not drift away from the leader, effectively setting the total energies of the two orbits equal. The resulting motion in the moving coordinate frame attached to the leader is an ellipse whose projection in the orbit plane is an ellipse having a semi-major axis twice its semi-minor axis. Furthermore, setting

$$\frac{2\dot{x}(0)}{\omega} = y(0),$$

puts the center of the ellipse at the leader, so that

$$x(t) = \frac{\dot{x}(0)}{\omega}\sin\omega t - \left(3x(0) + \frac{2\dot{y}(0)}{\omega}\right)\cos\omega t \quad (7)$$

$$y(t) = \frac{2\dot{x}(0)}{\omega}\cos\omega t + \left(6x(0) + \frac{4\dot{y}(0)}{\omega}\right)\sin\omega t \quad (8)$$

$$z(t) = \frac{\dot{z}(0)}{\omega}\sin\omega t + z(0)\cos\omega t. \quad (9)$$

It has been shown in [15] that the motion in general can be described as the intersection of a plane and an elliptic cylinder.

The relative motion equations (1)-(3) are for the unperturbed two-body motion. Here, we consider the performance of control strategies in the presence of the first earth gravity harmonic, the J_2 perturbation, as well as higher order gravity perturbations. The primary effects of J_2 are to rotate both the orbit plane and the orbit within the moving plane. The relative motion of closely orbiting satellites will vary from the Hill's equations solutions due to the differential perturbations on each of the satellites, and will tend to disperse the formation over time.

3 Formation Design

The relative motion equations derived in the previous section can be used to design formations for distributed satellites. In [8], the solutions to these equations were used to define four formations: in-track, in-plane, circular, and projected circular. In the first two, a leader satellite orbits ahead of a follower satellite. The in-track formation has a follower satellite in the same orbit while the in-plane formation has a follower in a slightly different orbit to to account for the rotation of the earth and give the two the same ground track. In the second two, a leader satellite is surrounded by an arbitrary number of follower satellites having the same orbital period. Small differences in inclination and ascending node crossings cause the follower satellites to appear to rotate in suborbits around the leader satellite in the moving coordinate frame attached to the leader. The circular formation has all of the follower satellites arranged in a circle while the projected circular formation has all of the followers in an ellipse whose projection onto the tangential plane is a circle. Both of these are special cases of the general elliptical shape of this type of formation.

We consider a projected circular formation of satellites. The leader satellite is assumed to be in a polar, circular orbit at 800km altitude. The projection of the formation's motion onto the earth's tangential plane is a one kilometer circle. A plot of the desired trajectory is shown in Figure 1 with projections onto each of the three planes. The initial orbital elements are given in Table 1 for the reference satellite and four satellites equally distributed around the ellipse. The angle θ is used to represent the initial phasing. Note that two of the satellites differ from the reference by their inclination i while the other two differ by their ascending node crossing Ω.

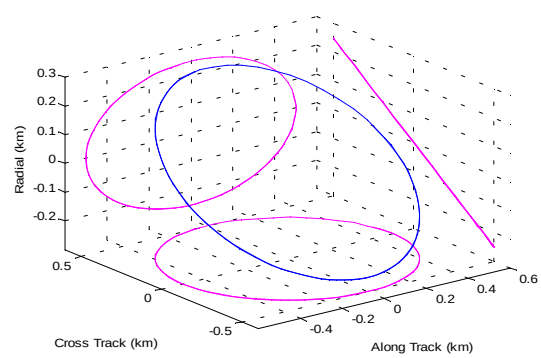

Figure 1: Trajectory of Projected Circular Formation

	Leader	Followers			
		$\theta = 0°$	$\theta = 90°$	$\theta = 180°$	$\theta = 270°$
$a(km)$	7178	7178	7178	7178	7178
e	0	3.48×10^{-5}	3.48×10^{-5}	3.48×10^{-5}	3.48×10^{-5}
$i(deg)$	90	90	90.004	90	89.996
$\Omega(deg)$	0	-0.004	0	0.004	0
$\omega(deg)$	0	180	270.016	0	89.984
$M(deg)$	0	180	89.984	0	270.016

Table 1: Orbital Elements of Formation

The projected circular formation is designed based on the linear solutions to the relative motion equations (7)-(9), and hence is an approximation to the true orbital dynamics. In particular, the nonlinearities and the orbital perturbations will lead to different relative motion dynamics than described by the linear equations. To see this, we can initialize the formation and perform nonlinear simulations with and without the J_2 gravity perturbation.

First, consider the unperturbed relative motion of the satellite with initial phasing $\theta = 0$ with no external disturbances or perturbations. The projected motion over one day is shown in Figure 2. The projected motion is as expected, since the nonlinearities for a one kilometer formation are very small. The errors in position come from the nonlinearity of the orbital dynamics, are small, and on average do not grow with time.

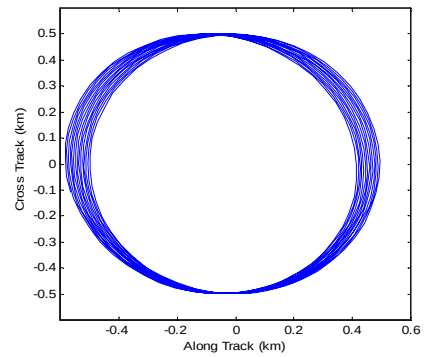

Figure 3: Projection of Motion with J_2

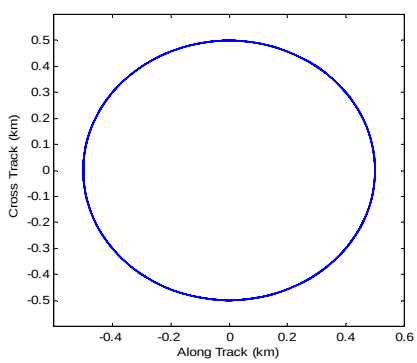

Figure 2: Projection of Unperturbed Motion

Next, consider the relative motion of the same satellite in the presence of the first gravity harmonic or J_2 perturbation. The projected motion over one day is shown in Figure 3. Even over a period of just one day the projected motion drifts noticeably and the errors in the relative motion are significant. Finally, the simulation was performed using a geopotential model to degree and order 21. The result was not perceptible different from the case of J_2 alone, confirming that it is the most significant perturbation. Note that the initial phase of the satellite in the ring around the leader will affect the motion. Specifically, the satellites whose ascending nodes are different from the leader behave differently from those whose inclinations are different. The result for either case is for the formation to disperse, and a control strategy will be required to maintain the relative geometry of the formation.

4 Formationkeeping Control

The projected circular formation defined in the previous section was derived using the linear relative motion equations for the two-body problem. Hence, for simple two-body orbital dynamics, the formation will be maintained indefinitely once the correct initial conditions are met. Unfortunately, gravity perturbations from earth asphericity, atmospheric drag, and solar radiation pressure all tend to perturb the orbits from their simple elliptical shapes predicted from two-body dynamics. Hence, control forces will have to be applied to maintain the relative distances of the formation.

Traditional satellite operations consist of long periods of uncontrolled behavior interrupted by small corrections to orbits. This can be effective since coarse satellite positioning requirements can be achieved through open loop behavior with small occasional corrections. However, formation flying will require much more demanding positioning requirements, and hence will require much more stringent control.

Unlike in most control problems, the application of forces to satellites to change their orbits can only be made over very short periods of time; the remainder of the time the satellite motion evolves naturally. This is because maneuvering propellant is scarce and cannot be replenished, chemical thrusters currently available as well more advanced propulsion systems such as pulse plasma thrusters are designed to work for limited periods, and thrusting tends to disrupt satellite operations. For these reasons, satellite control inputs can be modeled as impulsive changes to the satellites' velocities.

In this section, we apply state feedback, linear quadratic control to the formationkeeping problem described in the previous sections. Since the in-plane and out-of-plane motions are linearly decoupled, the feedback gains can be designed separately. The dynamic model of the relative motion (1)-(3) is written as a set of first order equations and discretized. In this case, to improve the accuracy of the model, the frequency of the satellite motion ω is corrected to account for the J_2 perturbation. The control inputs are chosen to be the impulsive velocity increments $(\Delta\dot{x}, \Delta\dot{y}, \Delta\dot{z})$ and the sampling time is chosen to be one hour. Hence, an impulsive velocity correction is computed and applied each hour. Using stan-

dard discrete-time LQ regulator theory, feedback gains are found to minimize the quadratic costs

$$
\begin{aligned}
J_{xy} &= \sum_{k=0}^{\infty} x^2(k) + y^2(k) + \rho \frac{\Delta V_x^2(k)}{\omega^2} + \rho \frac{\Delta V_y^2(k)}{\omega^2}, \\
J_z &= \sum_{k=0}^{\infty} z^2(k) + \rho \frac{\Delta V_z^2(k)}{\omega^2},
\end{aligned}
$$

where ρ is a parameter used to trade off the propellant usage and tracking accuracy of the control law. The resulting control laws are given by

$$
\begin{aligned}
\begin{bmatrix} \Delta \dot{x} \\ \Delta \dot{y} \end{bmatrix} &= -K_{xy} \begin{bmatrix} x - x_d \\ \dot{x} - \dot{x}_d \\ y - y_d \\ \dot{y} - \dot{y}_d \end{bmatrix}, \\
\Delta \dot{z} &= -K_z \begin{bmatrix} z - z_d \\ \dot{z} - \dot{z}_d \end{bmatrix},
\end{aligned}
$$

where (x_d, y_d, z_d) and $(\dot{x}_d, \dot{y}_d, \dot{z}_d)$ are the desired positions and velocities based on the linear, unperturbed relative motion dynamics.

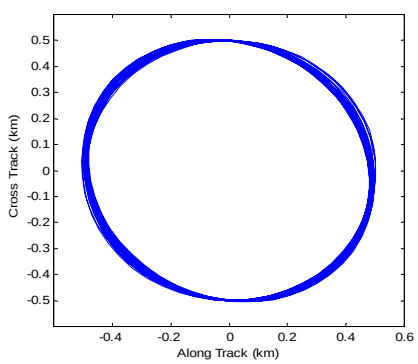

Figure 4: Projection of Motion with LQR and J_2

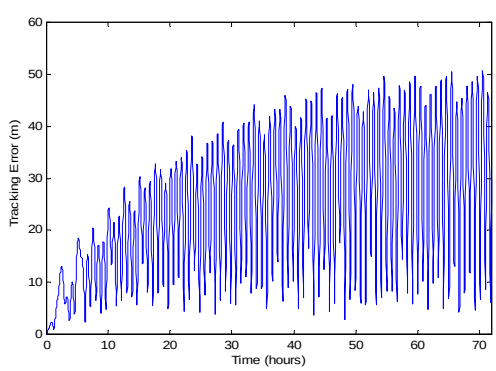

Figure 5: Tracking Error with LQR and J_2

The closed loop behavior is simulated using the LQ feedback gains and the orbital dynamics including the J_2 perturbation. The desired trajectory for the tracking problem is the solution (7)-(9), with ωt replaced by the argument of latitude of the leader satellite so that small errors in the orbital frequency do not create errors in the desired position that build up over time. The projected motion, tracking error, and impulsive control inputs are shown for the case of $\rho = 100$ for the satellite with initial phase angle of 90 degrees in Figures 4-6. The tracking error is limited to about 50 meters.

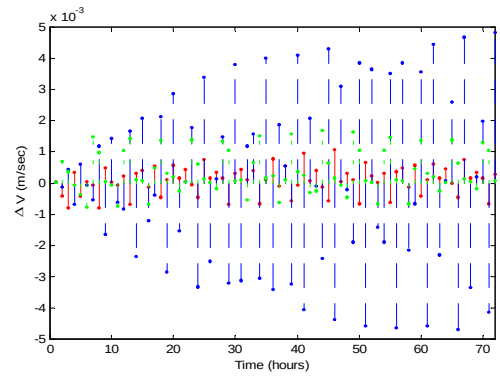

Figure 6: ΔV with LQR and J_2

To see the tradeoff between tracking error and fuel usage, the parameter ρ in the cost function is varied and the closed loop satellite behavior is again simulated including the J_2 perturbation, now for the satellite having different initial phase angles θ. Because of symmetry, the satellites having maximum node differences had similar performance, as did those having maximum inclination differences. In each case, the maximum errors and the ΔV per day required to maintain the formation were estimated using the simulation. The results are shown in Figures 7 and 8 and tabulated in Tables 2 and 3. Intuitively, as ρ increases the tracking error decreases while the amount of maneuvering propellant is reduced. In addition, the amount of propellant used to maintain the trajectories of the satellites with inclination differences ih higher than that required for the satellites with ascending node differences.

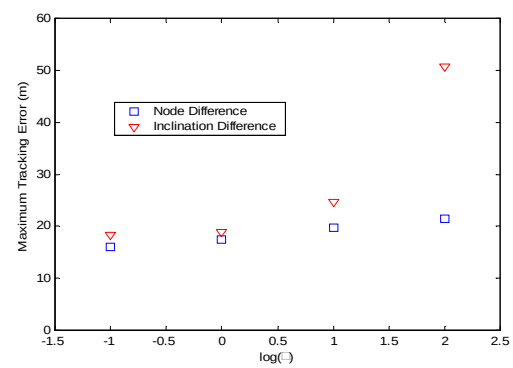

Figure 7: Maximum Tracking Error for Different ρ and θ

	$\rho = 0.1$	1	10	1000
$\theta = 0$	15.93	17.42	19.6	21.54
90	18.36	18.79	24.5	50.70
180	16.25	17.70	20.0	22.26
270	17.52	17.94	25.2	50.35

Table 1: Maximum Tracking Error for Different ρ and θ

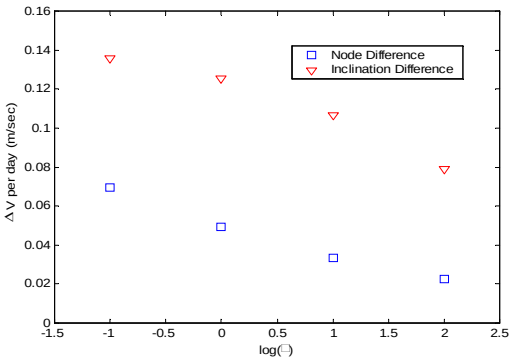

Figure 8: ΔV per day for Different ρ and θ

	$\rho = 0.1$	1	10	1000
$\theta = 0$	0.069	0.049	0.033	0.022
90	0.135	0.125	0.106	0.078
180	0.071	0.050	0.033	0.022
270	0.136	0.125	0.106	0.078

Table 2: ΔV per day for Different ρ and θ

Finally, the initial conditions of two satellites are displaced from their desired positions by placing them in the same circular orbit at the leader, but with a phase difference to put one approximately two and a half kilometers ahead of the leader and one the same distance behind. The control law is turned applied and both move toward their desired trajectories in the formation. A three dimensional plot of the motion is shown in Figure 9.

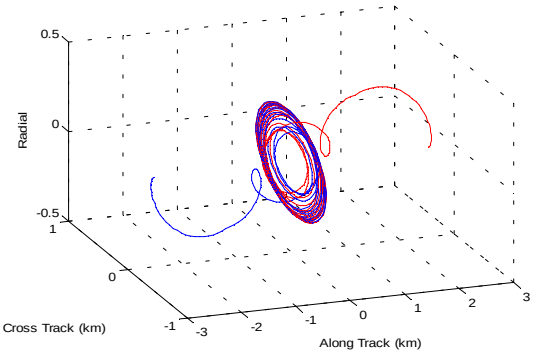

Figure 9: Offset Initial Condition Motion

5 Conclusions

In this paper, a feedback control strategy for formationkeeping of a projected circular formation of satellites was evaluated based on its ability to maintain a desired formation in the presence of earth gravity perturbations. A simple discrete-time linear quadratic regulator provided bounded error for the satellite formation. Simulations provided a baseline on the amount of formationkeeping propellant that would be required to maintain the formation.

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